IPPT Reports on Fundamental Technological Research, **47**, 4, 31–36, 2013
Institute of Fundamental Technological Research, Warszawa
Polish Academy of Sciences

SEMI-ACTIVE SUBOPTIMAL CONTROL STRATEGY APPLIED TO PERIODICALLY VIBRATING MECHANICAL SYSTEMS

Maciej MICHAJŁOW, Tomasz SZOLC

Institute of Fundamental Technological Research
Polish Academy of Sciences
Pawińskiego 5B, 02-106 Warszawa, Poland
e-mail: mmich@ippt.pan.pl

Received 18 November 2013, in final form 28 November 2013
Published online 28 November 2013

Abstract: The aim of this paper is to present the new method for obtaining the semi-active suboptimal control strategy, used for the purpose of vibration reduction in mechanical systems. The method is dedicated to systems where vibrations are of a periodic type. Generally, many semi-active control strategies have been developed up to date, but it seems that none of them are applied directly to the problem of mechanical systems suffering from disadvantageous, periodical vibrations. Probably, the reason why there were no such attempts in the past is that the semi-active vibration damping methods most often were used in situations where systems experience a finite time transient excitation, like for example seismic activity. In a case of systems experiencing repeatable (periodic), steady-state vibrations, such as machines or vehicles working at their nominal conditions, the common approach was to use the passive methods rather than the semi-active ones. This situation is changing nowadays and attempts of using semi-active methods to attenuate the disadvantageous steady state vibrations in machines and vehicles are becoming more common and successful. Therefore, there has been observed a surge in developing of the general purpose methods for determination of the optimal control strategies for such applications. The described method aspires to be the one of possible alternatives.

Key words: vibrations, semi-active damping, optimal control, magnetho-rheological damper.

1. Introduction

In mechanical engineering, especially in mechanical systems and vehicle dynamics, vibrations are usually considered as undesirable and damaging phenomenon. In order to reduce vibrations of a given mechanical system, it is necessary to add a concept of additional damping into the structure. There are at least three ways, in which such damping concepts can be realized in terms of their control, and those are: the passive, semi-active or active approach. Depending on the actual need, each path offers different possibilities. The passive approach is based on passive damping elements, such as common oil dampers. This approach is surely the easiest one to implement, as it requires no control at all. However, the damping effectiveness expected to be achievable in this way is rather low, comparing it to other options. The active approach is based on actively controlled elements, such as actuators. In this case the possibility of achieving good damping effectiveness is high. Nevertheless, the typical level of complexity of active damping system is also high and therefore this approach is often considered as expensive. In the semi-active case, the damping concept is usually realized by means of semi-actively controlled damping elements, such as dampers characterized by controllable damping coefficient. In the case of semi-active approach, the damping effectiveness as well as the level of damping device complexity is usually settled between both abovementioned damping concept options. Concluding, the semi-active approach can be considered as an intermediate solution. Such a moderate approach seems to have a potentially widest field of applicability. Therefore, the remaining part of this article will be focused on the methods for determination of the semi-active control strategies.

As it was already mentioned, the semi-active damping systems are essentially based on dampers with controllable damping coefficient. Probably, one of the most popular types of such dampers are the magneto-rheological dampers, or shortly MR dampers. The MR fluids have been discovered in the 1940s. Since then, many control methods dedicated to the MR dampers have been developed and verified in practice. Some examples are: the skyhook or groundhook control methods [1–3] (in the automotive branch) or the clipped-optimal control method [4] (in the seismic branch). Those kinds of methods have been developed with orientation on particular problems, such as a vehicle vibration reduction after running through the obstacle or, in the latter case, a vibration reduction in buildings experiencing seismic shocks. It is hard to expect that such methods could achieve the same good damping effectiveness when used in other applications, to which they were not originally dedicated to. Among other cases this applies to vibration reduction in machines and vehicles working at their nominal conditions (i.e., being affected by steady-state vibrations). There are of course other general control methods, which can be easily adapted to the MR dampers, for example, the optimization methods [5]. Nevertheless, the price for their universality lies in the high computational effort due to the typical large number of optimization arguments. The conclusion to the above mentioned observations is that there is still an open field for developing the effective control algorithms dedicated to the semi-active damping systems. In this paper the main attention will be focused on methods oriented to damping of periodical, steady-state vibrations in the mechanical systems.

2. The Suboptimal control method

The Suboptimal control method (SC method) is the novel semi-active method dedicated to mechanical systems experiencing disadvantageous, periodical vibrations, when working at their nominal conditions (steady-state). The easiest way to present the theoretical background of this method is by using the simple computational example. In this case, it will be the two degree of freedom (2-DOF) system presented in Fig. 1, composed of two masses m_1 and m_2 mutually connected by the linear spring of stiffness k_2 and by the MR damper generating damping force F_c. The assumed force-velocity dependency diagram of the damper (for the maximum control current), is shown in Fig. 1b. The lower mass m_1 is connected to the ground by means of the linear spring of stiffness k_1 and by the linear damper of damping coefficient c_1. The upper mass is excited by the harmonic force $F_e(t)$. This system is affected by vertical translational vibrations. It is assumed that no gravitational loadings are taken into consideration.

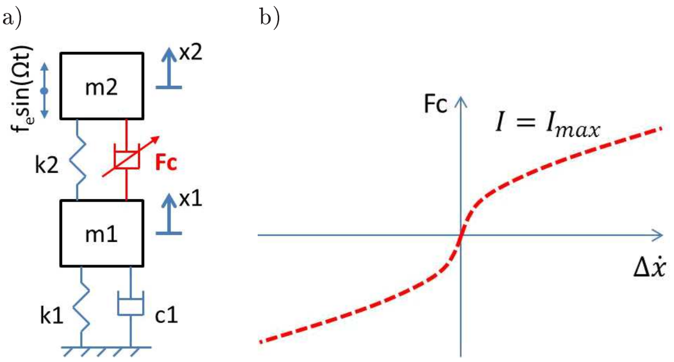

Fig. 1. a) The 2-DOF system layout, b) the force-velocity diagram of the MR damper.

The first step to be done in order to obtain the SC method is to virtually replace the MR damper with the actuator, see Fig. 2.

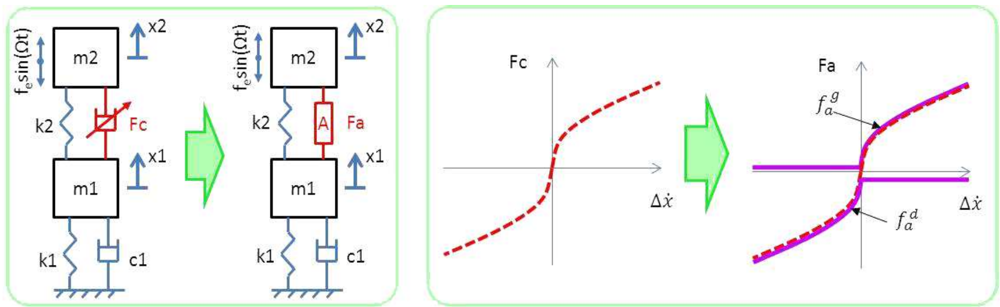

Fig. 2. The substitution of the MR damper with an actuator.

Afterwards, the following equations of motion can be derived:

$$(1)\quad \underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{M} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = -\underbrace{\begin{bmatrix} c_1 & 0 \\ 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \\ \cdot \underbrace{\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}}_{K} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \underbrace{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{\widehat{B}} f_a\left(\Delta\dot{x}\right) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\widehat{E}} f_e \sin\left(\Omega t\right),$$

where M, C, K are respectively the mass, damping and stiffness matrices, B, E are the vectors related to the places of force application and f_a denotes the actuator force.

As it has been shown in Fig. 2, the maximal and minimal force which actuator can generate is limited and depends on the current value of the relative velocity between both masses. The aim of a substituting the damper with the actuator is to isolate the damping force factor and nominate it as the bounded control function u:

$$(2)\qquad u = f_a\left(\Delta\dot{x}\right), \qquad u \in \langle f_z^d, f_a^g\rangle.$$

It is helpful to transform motion equations (1) together with substitution (2) into the state-space representation:

$$[\dot{q}] = \underbrace{\begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}}_{A} [q] + \underbrace{\begin{bmatrix} 0 \\ -M^{-1}\widehat{B} \end{bmatrix}}_{B} u + \underbrace{\begin{bmatrix} 0 \\ M^{-1}\widehat{E} \end{bmatrix}}_{E} f_e \sin(\Omega t)\,, \tag{3}$$

where q vector represents the current state of the system.

In order to reduce the vibrations in the considered system it is necessary to choose the measure of vibrations intensity. One of possible measures is the total mechanical energy of system. Because the system is affected by periodic motion, i.e., its displacements are systematically repeated at each time window $\langle 0, T\rangle$, it is to take the average energy over a single time-window. Therefore, the chosen vibration intensity measure has the following form:

$$J = 0.5 \int_0^T \left(q^T Q q + r u^2\right) dt, \qquad Q = \begin{bmatrix} K & 0 \\ 0 & M \end{bmatrix}, \qquad r \to 0. \tag{4}$$

Now, the problem of vibration reduction can be formulated mathematically as the problem of finding an extremum of the functional $\delta J = 0$. In this situation, u is the function on which functional J depends. This problem can be solved semi-numerically, i.e., using some additional transformations based on the optimal control theory. Its mathematical description reduces to the following relationships:

$$\begin{cases} \dot{q} = \dfrac{\partial H}{\partial \lambda} = Aq + Bu + E f_e \sin(\Omega t)\,, \\ \dot{\lambda} = -\dfrac{\partial H}{\partial q} = -Qq - A^T \lambda, \\ u = \begin{cases} f_a^g & \text{if} \quad B^T \lambda^* < 0, \\ f_a^d & \text{if} \quad B^T \lambda^* > 0, \end{cases} \end{cases} \tag{5}$$

where λ is the costate vector, which is coupled with the state vector q. The control function u has a characteristic of the bang-bang shape. The switching time-instants of u function depend on the state and costate vectors progress over the time window $\langle 0, T\rangle$. The boundary conditions are not entirely known and the only informations available are the following:

$$\lambda(T) = 0, \qquad q(T) = q(0). \tag{6}$$

Thus, the problem must be solved numerically using the following procedure:

1. Choose the initial guess for $q(T)$ vector.
2. Perform backward time integration of the system of equations, i.e., Eq. (5).
3. Examine, whether the periodicity constraint $(6)_2$ has been satisfied.
4. If the above constraint has not been satisfied, improve the initial guess, if else – finish the algorithm.

After a successful, i.e., a convergent run, the final u function shape and J value may be easily computed using Eqs. (4) and (5). The following Fig. 3 presents the results of a comparison between the time-history of the semi-active damping force and the passive damping force corresponding to the maximum possible constant control current. The resultant values of functional J have been also marked.

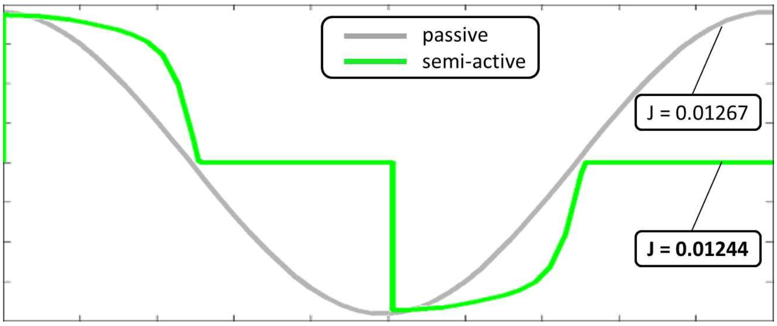

Fig. 3. The comparison of passive and semi-active damping force time-histories over the single time window $\langle 0, T\rangle$.

3. Conclusions

This paper presents the method for determination of the suboptimal control strategy used for attenuation of vibrations in periodically oscillating mechanical systems. The mathematical background of this method has been developed on the basis of optimal control theory using the additional periodicity constraint. The exact solution in the form of suboptimal control strategy is obtained by means of numerical computations. It has been shown that semi-active, suboptimal control strategy can lead to better suppression of vibrations than the passive damping approach. In general, this may not be true because this method strongly depends on the initial guess. Although this has not been shown in the presented paper, the considered suboptimal control method typically requires less optimization arguments than the common optimization methods used for the purpose of attenuation of vibrations in the considered class of problems. It is important to emphasize that the suboptimal control strategy can give comparable results of vibration reduction, even with a lower computational effort, in a comparison with selected well- known optimization methods.

Acknowledgments

Financial assistance of the NCN, project contract no. UMO-2012/07/N/ST-8/03300, is acknowledged.

References

1. D. Karnopp, *Active and semi-active vibration isolation*, ASME J.Mech. Des. **117**, 177–185, 1995.
2. D. Karnopp, M.J. Crosby, R.A. Harwood, *Vibration controlusing the semi-active force generators*, ASME J. Eng. Ind. **96**, 619–626, 1974.

3. M. Novak, M. Valasek, *A new concept of semi-active control of trucks suspension*, in Proceedings of AVEC 96, International Symposium on Advanced Vehicle Control, Aachen University of Technology, pp. 141–151, 1996.

4. S.J. Dyke, B.F. Spencer Jr., M.K. Sain, J.D. Carlson, *Seismic response reduction using magnetorheological dampers*, Proc. of the IFAC World Congress, San Francisco, California, June 30 – July 5, 1996.

5. A. Pręgowska, *Semi-active control of the torsional vibrations, of the machine drive system, using intelligent dampers with magnetho-rheological fluid, Pół-aktywne sterowanie drganiami skrętnymi układu napędu maszyny roboczej przy wykorzystaniu inteligentnych tłumików z cieczą magnetoreologiczną* [in Polish], Ph.D. Thesis., IPPT PAN, 2013.